DELTA MATH SCIENCE PARTNERSHIP INITIATIVE

M³ Summer Institutes

(Math, Middle School, MS Common Core)

Place Value and Fractions

Fractions most commonly are represented as part to whole. Keep this in mind. What does "place value" mean? Is our number system a place value system?

A place value system has a base. What is the base of our system?

thousands hundreds tens ones

In elementary school, the system is expanded to the "left" for greater place value. Can it be extended in the other (the "right") direction?

thousands	hundreds	tens	ones	tenths	hundredths	thousandths
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
$\frac{10,000}{10}$	$\frac{1,000}{10}$	$\frac{100}{10}$	$\frac{10}{10}$	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1,000}$

As one moves from left to right, we subtract one from the power each time, which means divide by 10. This produces the "common fractions" that have powers of ten in the denominators.

Use of common fractions came into common use in the late 1500's and early 1600's (Simon Stevin). Before this, all arithmetic was done using fractions.

Write in expanded place value form

$$246 = 2(10^2) + 4(10) + 6(10^0)$$
$$= 2(100) + 4(10) + 6(1)$$

This is easy for whole numbers. Now,

$$12.469 = 1(10^1) + 2(10^0) + 4(10^{-1}) + 6(10^{-2}) + 9(10^{-3})$$
$$= 1(10) + 2(1) + 4(\frac{1}{10}) + 6(\frac{1}{100}) + 9(\frac{1}{1000})$$

or

$$= 1(10) + 2(1) + \frac{4}{10} + \frac{6}{100} + \frac{9}{1000}$$

In order to keep the place value, one does not simplify any fraction.

Change 2.56 into fraction form.

$$2.56 = 2 + \frac{5}{10} + \frac{6}{100}$$
$$= \frac{200}{100} + \frac{50}{100} + \frac{6}{100} = \frac{256}{100}$$

Notice that the last place value defines the denominator.
Add 2.46+1.7 using fraction form.

$$2.46 + 1.7 = 2 + \frac{4}{10} + \frac{6}{100} + 1 + \frac{7}{10}$$ (add fractions with like denominators)

$2 + \frac{4}{10} + \frac{6}{100}$

$1 + \frac{7}{10}$

(use this to form a general rule for adding without changing to fraction form)

Would subtraction work the same way as addition? Make students write a conjecture as to how this should work. Refrain from giving the students a rule – make them come to the rule on their own.

What does multiplication of decimals become if we convert to decimal-fractions?

$$(2.46)(0.18) = ?$$
$$= \frac{246}{100} \times \frac{18}{100}$$
$$= \frac{4,428}{10,000}$$

Now change back to decimal fractions.

$$= 0.4428$$ (Notice: 246 x 18 = 4,428)

What is (24.6)(0.18) = ?

$$\left(\frac{246}{10}\right)\left(\frac{18}{100}\right)=\frac{4,428}{1,000}$$

convert:

$$=4.428$$

Keep the digits the same in order to explore the behavior of the decimal point.

Begin with 37 x 502 = 18,574. Use this fact and write the following division example:

$$37\overline{)18,574}\;502$$

$$=\frac{18,574}{37}$$

$$=502$$

Answer:

$$\left[\frac{18,574}{37}\right]\left[\frac{5}{5}\right]=?$$

(fancy way of writing 1)

Can 1 be written as $\left[\dfrac{\frac{1}{100}}{\frac{1}{100}}\right]$?

What is $\dfrac{18,574}{37}\dfrac{\left(\frac{1}{100}\right)}{\left(\frac{1}{100}\right)}=\dfrac{185.74}{0.37}$

$$=502$$

True or false: 112 ÷ 8 = 14?

2 x 112 = 224 so 224 ÷ 8 = _____
5 x 112 = 560 so 560 ÷ 8 = _____
½ x 112 = 56 so 56 ÷ 8 = _____

2 x 8 = 16 so 112 ÷ 16 = _____

½ x 8 = 4 so 112 ÷ 4 = _____

Conjecture: if the dividend remains the same and the divisor is multiplied by a non-zero number k, then the quotient _____.

if the divisor remains the same and the dividend is multiplied by a non-zero number k, then the quotient _____.

if both the dividend and divisor are multiplied by a non-zero number k, then the quotient _____.

What does percent mean?

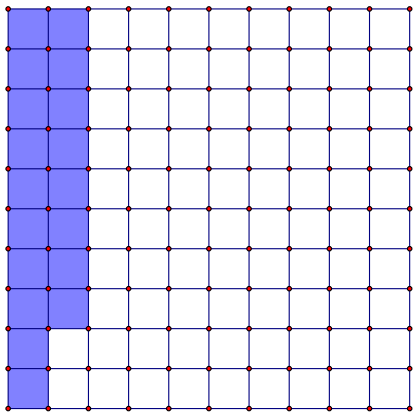
Parts out of one-hundred i.e., one wishes to have a denominator of 100.

For example, 18% means $\frac{18}{100}$ and 72% means $\frac{72}{100}$.

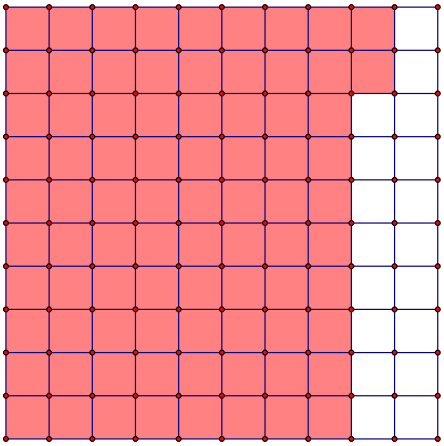
Likewise, $\frac{52}{100}$ means 52% and $\frac{63}{100}$ means 63%.

A hundreds grid therefore represents a whole so 18% or $\frac{18}{100}$ means

18 squares out of 100



Thus, 82% would look like



What does this mean?